CAMBRIDGE INTERNATIONAL EXAMINATIONS

Cambridge Ordinary Level

MARK SCHEME for the October/November 2015 series

4037 ADDITIONAL MATHEMATICS

4037/23 Paper 2, maximum raw mark 80

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CAMBRIDGE
International Examinations

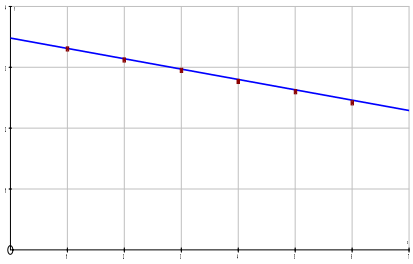
Abbreviations

awrt answers which round to
cao correct answer only
dep dependent
FT follow through after error
isw ignore subsequent working
oe or equivalent
rot rounded or truncated
SC Special Case
soi seen or implied
www without wrong working

1	$y = x^3 + 3x^2 - 5x - 7$ $\frac{dy}{dx} = 3x^2 + 6x - 5$ $x = 2 \rightarrow \frac{dy}{dx} = 19$ $y = 3$ eqn of tangent: $\frac{y-3}{x-2} = 19 \quad \rightarrow \quad (y = 19x - 35)$	M1 A1 A1FT B1 A1FT	Differentiate on *their* $\frac{dy}{dx}$
2	$2x + k + 2 = 2x^2 + (k+2)x + 8$ $2x^2 + kx + 6 - k \quad (= 0)$ $b^2 - 4ac = k^2 - 4 \times 2(6-k)$ $k^2 + 8k - 48 \quad (> 0)$ $(k+12)(k-4) \quad (> 0)$ $k < -12$ or $k > 4$	M1 A1 M1 DM1 A1 A1	eliminate y or x correct quadratic use discriminant attempt to solve 3 term quadratic $k = -12$ and $k = 4$
3 (a)	$\frac{dy}{dx} = \frac{(2-x^2)3x^2 - x^3(-2x)}{(2-x^2)^2} = \left(\frac{6x^2 - x^4}{(2-x^2)^2}\right)$	M1 A2,1,0	For quotient rule (or product rule on correct y)
(b)	$\frac{dy}{dx} = x \times \frac{1}{2}(4x+6)^{-0.5} \times 4 + (4x+6)^{0.5}$ $= \frac{6(x+1)}{(4x+6)^{0.5}} \quad \rightarrow \quad k = 6$	M1 A1 A1	product rule
4	$x(4 - \sqrt{3}) = 13$ $x = \frac{13\left(4+\sqrt{3}\right)}{\left(4-\sqrt{3}\right)\left(4+\sqrt{3}\right)}$ $= 4 + \sqrt{3}$ $y = 1 - 2\sqrt{3}$	M1 A1 M1 A1 A1	eliminate y or x simplified rationalisation

5	$(x-3)(x-3)(x-1)=0$	M1	
	$x^3-7x^2+15x-9=0$		
	$a=-7$	A1	
	$b=15$	A1	
	$c=-9$	A1	**AG** for c
6	$\log_x 2=\dfrac{\log_2 2}{\log_2 x}$	B1	
	$2\log_2 x=\log_2 x^2$	B1	
	$3=\log_2 8$	B1	
	$8x^2-29x+15\ (=0)$	M1	obtain quadratic and attempt to solve
	$\rightarrow (8x-5)(x-3)\ (=0)$		
	$x=\dfrac{5}{8}$ or $x=3$	A1	
7 **(i)**	$a=-\dfrac{20}{(t+2)^3}$	M1 A1	$k(t+2)^{-3}$ oe $k=-20$
	$t=3 \rightarrow a=-0.16$ m/s^2	A1FT	
(ii)	$\dfrac{10}{(t+2)^2}$ is never zero.	B1	
(iii)	$s=-\dfrac{10}{t+2}+5$	M1 A1 A1	integrate $\dfrac{k}{t+2}$ $k=-10$ +5
(iv)	$s=\left[-\dfrac{10}{t+2}\right]_3^8=-1+2$	M1	insert limits and subtract
	$=1$	A1	

8	**(i)**	$\sec^2 x + \text{cosec}^2 x = \dfrac{1}{\cos^2 x} + \dfrac{1}{\sin^2 x}$	B1	
		$= \dfrac{\sin^2 x + \cos^2 x}{\sin^2 x \cos^2 x}$	B1	add fractions
		$= \dfrac{1}{\sin^2 x \cos^2 x}$	B1	use of $\sin^2 x + \cos^2 x = 1$
		$= \sec^2 x \text{cosec}^2 x$	B1	fully correct solution
	(ii)	$\dfrac{1}{\cos^2 x \sin^2 x} = 4\dfrac{\sin^2 x}{\cos^2 x}$	M1	
		$\rightarrow \quad 4\sin^2 x = 1$	A1	correct simplified equation
		$\sin x = \pm\dfrac{1}{\sqrt{2}}$		
		$x = 135°, 225°$	A1, A1	
9	**(i)**	$\text{f}(x) = 3x^2 + 12x + 2 = 3(x+2)^2 - 10$		
		$a = 3$	B1	
		$b = 2$	B1	
		$c = -10$	B1	
	(ii)	minimum $\text{f}(x) = -10$	B1FT	
		at $x = -2$	B1FT	
	(iii)	$\text{f}\left(\dfrac{1}{y}\right) = 0 \quad \rightarrow \quad \left(\dfrac{1}{y}\right) = (\pm)\sqrt{\dfrac{10}{3}} - 2$	M1	obtain explicit expression for $\dfrac{1}{y}$ or y
		$y = -5.74, \quad -0.26$	A1, A1	

10	**(i)**	$\frac{\mathrm{d}}{\mathrm{d}x}(\mathrm{e}^{2-x^2}) = -2x\mathrm{e}^{2-x^2}$	B1	$k = -2$
	(ii)	$-\frac{3\mathrm{e}^{2-x^2}}{2} + c$	M1 A1FT	$D\mathrm{e}^{2-x^2}$ $D = \frac{-3}{2}$ or $\frac{3}{k}$
	(iii)	$\left[-\frac{3\mathrm{e}^{2-x^2}}{2}\right]_1^{\sqrt{2}} = -\frac{3}{2} + \frac{3}{2}\mathrm{e}$ 2.58	M1 A1	insert limits on *their* (ii) and subtract
	(iv)	$y = 3x\mathrm{e}^{2-x^2}$ $\frac{\mathrm{d}y}{\mathrm{d}x} = 3x(-2x\mathrm{e}^{2-x^2}) + 3\mathrm{e}^{2-x^2}$ $\frac{\mathrm{d}y}{\mathrm{d}x} = 0 \rightarrow x = \pm\frac{1}{\sqrt{2}} = \pm 0.707$ $y = \pm\frac{3}{\sqrt{2}}\mathrm{e}^{1.5} = \pm 9.51$	M1 A1 A1 A1	product rule both x or a pair both y
11	**(i)**	$\log N = \log A - t\log b$	B1	
	(ii)	t: 1 2 3 4 5 6 log N: 3.30 3.11 2.95 2.77 2.60 2.41 ln N: 7.60 7.17 6.79 6.38 5.98 5.56 	M1 M1 A1	find logs of N plot log$N$ or ln$N$ against t or $-t$ straight line passing through five points
	(iii)	gradient $= -\log b = \frac{2.415 - 3.3}{5} \rightarrow b = 1.5$ intercept $= \log A = 3.47 \rightarrow A = 2950$	DM1 DM1 A1	set gradient = $-\log b$ and solve set intercept = $\log A$ and solve both values correct
	(iv)	$t = 10 \rightarrow N = \frac{2950}{1.5^{10}} = 51$	B1	
	(v)	$N = 10 \rightarrow 1.5^t = 295 \rightarrow t = \frac{\log 295}{\log 1.5}$ $= 14$ years	M1 A1	substitute $N = 10$, *their* A, b into given or transformed equation

12	$v_p = \begin{pmatrix} 250\cos 20^\circ \\ 250\sin 20^\circ \end{pmatrix}$, $v_r = \begin{pmatrix} V\cos 30^\circ \\ V\sin 30^\circ \end{pmatrix}$, $v_w = \begin{pmatrix} 0 \\ w \end{pmatrix}$	B1	
	$v_r = v_p + v_w$ $\begin{pmatrix} V\cos 30^\circ \\ V\sin 30^\circ \end{pmatrix} = \begin{pmatrix} 250\cos 20^\circ \\ 250\sin 20^\circ \end{pmatrix} + \begin{pmatrix} 0 \\ w \end{pmatrix}$		
	$V = \dfrac{250\cos 20^\circ}{\cos 30^\circ}$ $= 271\text{km/hr}$	M1 A1	equate x components and solve
	$w = V\sin 30^\circ - 250\sin 20^\circ$ $= 50.1\text{km/hr}$	M1 A1	equate y components and solve
	OR triangle with sides 250 V w opposite angles 60° 110° 10°	B1	
	sine rule: $\dfrac{w}{\sin 10^\circ} = \dfrac{250}{\sin 60^\circ}$ $w = 50.1\text{km/hr}$	M1 A1	apply to correct triangle and solve
	$\dfrac{V}{\sin 110^\circ} = \dfrac{250}{\sin 60^\circ}$ $V = 271\text{km/hr}$	M1 A1	apply to correct triangle and solve